Int. J. Heat Mass Transfer. Vol. 24, No. 6, pp. 1071–1075, 1981
Printed in Great Britain

0017-9310/81/061071-05 $02.00/0
Pergamon Press Ltd.

SHORTER COMMUNICATIONS

LAMINAR FREE CONVECTION ON AN INCLINED FLAT PLATE OR A VERTICAL CYLINDER WITH PRESCRIBED WALL HEAT FLUX

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(*Received* 26 *May* 1980 *and in revised form* 29 *October* 1980)

NOMENCLATURE

C_τ, C_{Nu}, c,	constants;
F,	dimensionless stream function;
g,	acceleration due to gravity;
Gr^E,	local flux Grashof number ($= g_x\beta q_w L^4/kv^2$), dimensionless;
k,	thermal conductivity coefficient;
L,	characteristic length;
N,	coefficient defined in [4];
Nu,	local Nusselt number $[= q_w x/k(T_w - T_\infty)]$, dimensionless;
p,	dimensionless reduced coordinate;
Pr,	Prandtl number $(= v/\alpha)$, dimensionless;
q,	heat flux;
Q,	amount of energy convected by the fluid;
t,	normalizing arbitrary constant;
T,	fluid temperature;
u, v,	velocity components;
x, y,	tangential and normal coordinates respectively.

Greek symbols

α,	thermal diffusivity coefficient;
β,	thermal expansion coefficient;
γ,	angle between the plate and the vertical direction;
ε,	dimensionless parameter function;
λ,	constant;
v,	fluid viscosity;
ρ,	fluid density;
τ,	skin friction;
η,	dimensionless reduced coordinate;
ω, σ, φ,	dimensionless functions;
ϕ,	dimensionless stream function;
θ,	reduced fluid temperature.

Subscripts

Ca,	results from [4];
Ge,	results from [1];
Ka,	results from [5–7];
n,	series order;
ref,	numerical method's results given in [5–7];
Sp,	results from [2]
w,	at wall surface;
x,	in the x-direction;
Z,	author's results or functions;
∞,	at bulk conditions;
0,	dummy integration variable.

Superscripts

$\hat{}$,	dimensional variables;
$\bar{}$,	averaged along the wall.

Operators

∂,	derivated with respect to the subscript(s).

INTRODUCTION

LAMINAR boundary-layer natural-convection on a vertical wall with uniform wall heat flux has been thoroughly treated by several authors, mainly in [1–3]. Sparrow [3], proposed an approximated method to treat the nonuniform wall heat flux case. This method has been criticized by several authors [5–9] where some local nonsimilar methods are proposed.

Natural-convection flow near an inclined flat surface was first visualized by Schlieren. Rich [10] measured heat transfer at different inclination angles and found it within 10% of the values for the vertical plate if Gr is multiplied by $\cos(\gamma)$. Vliet [11] and Black and Norris [16] supported Rich's suggestion.

Kierkus [17] used a perturbation method to solve the boundary layer equations for isothermal plates with inclinations between $-45°$ and $+45°$. Lee and Lock [18] solved this problem numerically for air and for inclination angles from 90° to $-30°$.

The instability of such flows was studied by Lloyd, Sparrow and Eckert [13] and by several other authors [18–38], where some parameters are proposed, indicating the beginning of the transition region and its dependence on the inclination angle, the local heat flux, the Prandtl number and etc.

The mathematical model describing natural convection flows around vertical cylinder is analogous to that corresponding to the flat plate, thus the solution is the same for both geometries.

It is the aim of this paper to propose a new nonsimilar solution method to the nonuniform wall heat flux boundary-layer on inclined plates and vertical cylinder, easy to handle and applicable in all the range of laminar flow, whose limits can be determined by experimental or appropriate theoretical study.

The case of binomial $[q_0(1 + \lambda x)^\varepsilon]$ and exponential $(q_0 e^{\lambda x})$ wall heat flux distributions is solved for $Pr = 0.7$. The solution is given in the form of diagrams giving the distribution of a reduced skin friction and a reduced fluid temperature, *viz.* (λx).

BOUNDARY-LAYER EQUATIONS AND TRANSFORMATIONS

In the case of laminar free convection along an inclined flat surface or a vertical cylinder with constant properties and prescribed steady wall heat flux, introducing the following dimensionless variables

$$x = \frac{\hat{x}}{L}, \qquad y = \frac{\hat{y}}{L} \tag{1}$$

$$u = \frac{L}{\nu}\hat{u}, \qquad v = \frac{L}{\nu}\hat{v} \tag{2}$$

$$T = \beta(\hat{T} - \hat{T}_\infty), \quad Gr^E_{(x)} = \frac{\beta L}{K} q_{w(x)} \tag{3}$$

$$L = \left(\frac{\nu^2}{g_x}\right)^{1/3} \tag{4}$$

the conservation equations are given by

$$\partial_x u + \partial_y v = 0 \tag{5}$$

$$u\partial_x u + v\partial_y u = \partial_{yy} u + T \tag{6}$$

$$u\partial_x T + v\partial_y T = \frac{1}{Pr}\partial_{yy} T \tag{7}$$

with the following boundary conditions

at

$$y = 0, \quad u = 0 = v, \quad \partial_y T = Gr^E \tag{8}$$

at

$$y \to \infty, \quad u \to 0, \quad T \to 0. \tag{9}$$

Let us introduce the following transformations

$$u = \partial_y \phi, \quad v = -\partial_x \phi \tag{10}$$

$$\varphi = (1 + \lambda x) \tag{11}$$

$$\sigma = (Gr^E_{(\varphi)})^{2/3}, \quad \omega = \int_0^\varphi \sigma \, \mathrm{d}\varphi_0 \tag{12}$$

$$p = \frac{1}{t}\frac{\omega\partial_\varphi\sigma}{\sigma^2}, \quad t \geq 2\left[\frac{\omega\partial_\varphi\sigma}{\sigma^2}\right]_{\max} \tag{13}$$

$$\eta = \frac{\sigma^{0.5}}{\omega^{0.2}}y, \quad \varepsilon = \frac{\sigma\partial_{\varphi\varphi}\sigma}{(\partial_\varphi\sigma)^2} \tag{14}$$

$$F_{(p,\eta)} = \frac{\sigma^{0.5}}{\omega^{0.8}}\phi, \quad \theta_{(p,\eta)} = \frac{1}{\sigma\omega^{0.2}}T. \tag{15}$$

The resulting reduced partial differential equations' system is

$$\partial_{\eta\eta\eta}F = 0.6(\partial_\eta F)^2 + (0.5\,tp - 0.8)F\partial_{\eta\eta}F - \theta + [p + t(\varepsilon - 2)p^2](\partial_\eta F\partial_{p\eta}F - \partial_p F\partial_{\eta\eta}F) \tag{16}$$

$$\partial_{\eta\eta}\theta/Pr = (0.2 + tp)\theta\partial_\eta F + (0.5\,tp - 0.8)F\partial_\eta\theta + (p + t(\varepsilon - 2)p^2)(\partial_\eta F\partial_p\theta - \partial_p F\partial_\eta\theta) \tag{17}$$

at

$$\eta = 0, \quad F = \partial_\eta F = \partial_\eta\theta + 1 = 0 \tag{18}$$

at

$$\eta \to \infty, \quad \partial_\eta F \to 0, \quad \theta \to 0. \tag{19}$$

SOLUTION METHODS

Systems (16)–(19) can be solved by different methods, such as the finite element method, finite difference method, etc. Due to the time and cost of computing of the first two methods, we chose the following series expansion method

let

$$\varepsilon = \sum_{n=0}^{\infty} \varepsilon_n p^n \tag{20}$$

$$F = \sum_{n=0}^{\infty} F_{n(\eta)} p^n \tag{21}$$

$$\theta = \sum_{n=0}^{\infty} \theta_{n(\eta)} p^n \tag{22}$$

the choice of the arbitrary constant t given by equation (13) is such that $p < 0.5$ to avoid the divergence of these series.

Substituting in equations (16–19) and separating the equal powers of p, we obtain a set of recurrent coupled pairs of ordinary differential equations. The first pair (the zeroth-order) is nonlinear and corresponds to the uniform wall heat flux distribution; while the higher orders are linear and corresponds to the corrections due to the prescribed distribution.

RESULTS AND COMPARISON

(1) *Uniform flux distribution* ($p = 0$, $\varepsilon = 0$)

Analysis shows that

$$\tau_w = C_\tau, \quad \partial_{\eta\eta}F_0\, x^{0.4} \tag{23}$$

$$Nu = \frac{1}{\theta_0} C_{Nu}\, x^{0.8} \tag{24}$$

$$C_\tau = \rho\left[\left(\frac{g\beta q_w}{k}\right)^3 \nu^4\right]^{1/5} \tag{25}$$

$$C_{Nu} = \left(\frac{g\beta q_w}{k\nu^2}\right)^{1/5}. \tag{26}$$

Mahajan and Gebhart [1] and Sparrow and Gregg [2] examined the same case. To simplify the comparison, we can write their results as

$$\tau_{w,Ge,0} = C_\tau(5^{2/5} f_0'')_{Ge} \quad \hat{x}^{0.4} = C_\tau V_{Ge}\hat{x}^{0.4} \tag{27}$$

$$Nu_{Ge,0} = \frac{C_{Nu}}{(5^{1/5}\,T_0)_{Ge}} \quad \hat{x}^{0.8} = \frac{C_{Nu}}{\theta_{Ge}}\hat{x}^{0.8} \tag{28}$$

$$Nu_{Sp} = \frac{C_{Nu}}{(5^{0.2}\,\theta_0)_{Sp}} \quad \hat{x}^{0.8} = \frac{C_{Nu}}{\theta_{Sp}}\hat{x}^{0.8} \tag{29}$$

where the subscripts Ge and Sp and the variables with parentheses refer to [1] and [2], respectively. $\tau_{w,Ge,0}$ and

Table 1. Comparison between authors' results and results given by [1] and [2] for the uniform flux distribution

Pr	θ_{OZ}	θ_{Ge}	θ_{Sp}	$\partial_{\eta\eta}F_0$	V_{Ge}
0.1	3.7949	—	3.9910	3.1256	—
0.733	2.0414	2.0417	—	1.5392	1.5399
1.0	1.8726	—	1.8250	1.3738	—
6.7	1.1615	1.1613	—	0.6781	0.6783

Table 2. Comparison with experimental results [4]

Fluid	Pr	N	θ_{Ca}	θ_Z
Air, 52°C	0.703	39.47	2.0611	2.0657
Water, 20°C	7.060	3.62	1.139	1.147

$Nu_{Ge,0}$ denote the zeroth-order Gebhart variables.

Table 1 compares our results $[\theta_0, (\partial_{\eta\eta}F_0)]$ with V_{Ge}, θ_{Ge} and θ_{Sp}.

To enable comparison with the experimental results published in [4], a dimensionless reduced-temperature function corresponding to these results was found to be given by

$$\theta_{Ca} = \left(\frac{K^4\beta g}{v^2 q}\right)^{0.2} N. \tag{30}$$

Table 2 compares θ_Z found by our method and θ_{Ca} calculated by equation (29).

(2) *Non-uniform flux distribution*

Kao [5–7] gives a local nonsimilar solution method for these cases. Mainly he solves cases of linear increasing, linear decreasing and exponential increasing flux distributions. He also compares his results with results obtained from the local similarity model used in [8] and [9] and those obtained with the most accurate difference–differential method. The latest results will be considered when comparing our results with Kao's.

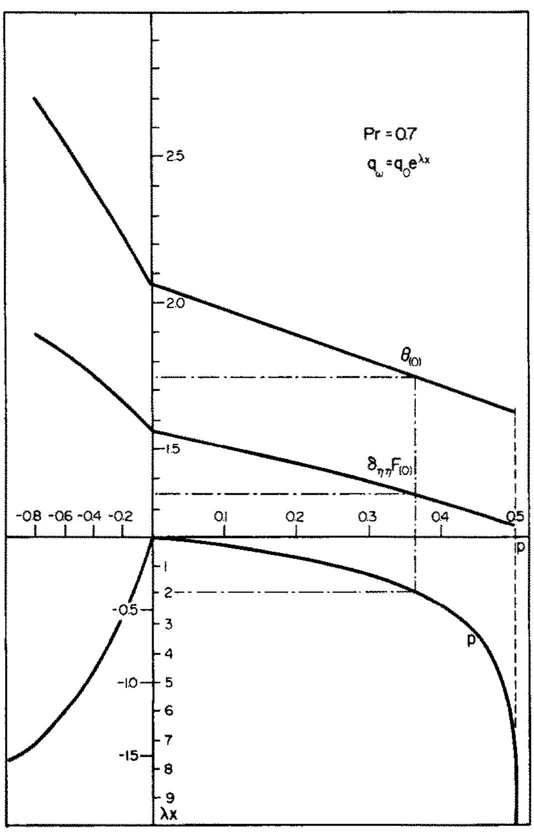

FIG. 1. Diagram for p, $\theta_{(0)}$ and $f''_{(0)}$ vs λx for an exponentially varying wall heat flux. –·–·–·– for $\lambda x = 2$; – – – – asymptotic line, valid for large values of λx.

Applying the method described in this paper to the problem of exponential flux distribution

$$q_w = e^{\lambda x} \tag{31}$$

we get

$$\varepsilon_0 = 1, \quad \varepsilon_{n>0} = 0. \tag{32}$$

The resulting set of total differential equations was solved numerically using the fourth-order-Runge–Kutta method for $Pr = 0.7$ taking arbitrarily $t = 2$.

Figure 1 shows a diagram from which one can find $p_{(\lambda x)}$, $\partial_{\eta\eta}F_{(0,\lambda x)}$ and $\theta_{(0,\lambda x)}$. The method is as follows knowing λx, locate it on the vertical axis and draw a horizontal line (to the right-hand side for positive λx and to the left-hand side for negative λx), from the intersection of this line with the p-curve draw a vertical line that will cut the $\partial_{\eta\eta}F$ and the θ curve at their respective corresponding values; an example is drawn for $\lambda x = 2$. An asymptotic vertical line is also drawn, corresponding for large values of λx.

Note that

$$Nu = \frac{Gr^E}{\sigma\omega^{0.2}\theta_{(0)}} \tag{33}$$

$$\tau_w = (\rho\, gL)\,\sigma^{0.5}\,\omega^{0.4}\,\partial_{\eta\eta}F_{(0)}. \tag{34}$$

Table 3 compares the dimensionless wall temperature distributions that we obtained (T_Z), with those of Kao (T_{Ka}) and the reference method (T_{ref}) for $q = e^x$.

For the binomial flux distribution

$$q_w = q_0(1+\lambda x)^c \tag{35}$$

we have

$$\varepsilon_0 = \frac{c-1.5}{c}, \quad \varepsilon_{n>0} = 0. \tag{36}$$

This case was also solved for $Pr = 0.7$, $t = 2$ and $c = 1$.

Figure 2 gives the corresponding diagram between $\theta_{(0,\lambda x)}$, $\partial_{\eta\eta}F_{(0,\lambda x)}$, $p_{(\lambda x)}$ and (λx). An example is drawn for $\lambda x = 2$ and the asymptotic vertical line at $p = 0.2$ correspond to the power-law distribution.

Table 4 compares T_Z, T_{Ka} and T_{ref} for $q = 1 + x$.

Figure 3 shows the variation of $Nu_{(x)}$ and T viz. $Q_{(x)}$ for the linear and the exponential flux distribution, where

Table 3. Comparison for $q = e^x$

x	$T_{Z(0)}$	$T_{ref(0)}$	$T_{Ka(0)}$
0	0	0	0
0.2	1.6922	1.6911	1.6911
0.4	2.1994	2.1980	2.2022
0.6	2.7065	2.7047	2.7119
0.8	3.2610	3.2588	3.2720
1.0	3.8870	3.8843	3.9030
1.4	5.4349	5.4311	5.4502
2.0	8.8369	8.8176	8.8498
2.4	12.1690	12.1420	12.1870

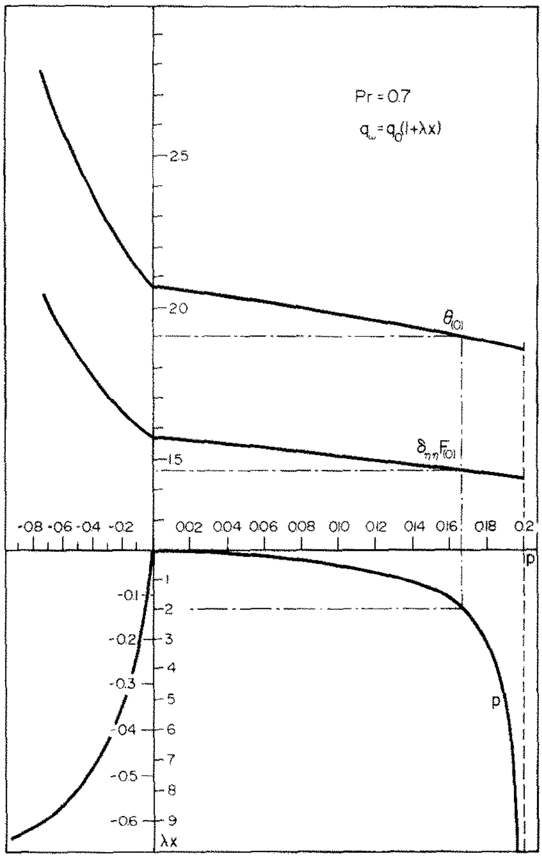

FIG. 2. Diagram for p, $\theta_{(0)}$ and $f''_{(0)}$ vs λx for a linearly varying wall heat flux $-\cdot-\cdot-$ for $\lambda x = 2$; $----$ asymptotic line, valid for large λx and for $q_w = ax^{\varepsilon}$.

$$\overline{Nu}_{(x)} = \frac{Q_{(x)}}{\overline{T}_{(x)}} \tag{37}$$

$$Q_{(x)} = \int_0^x q_w \, dx_0 \tag{38}$$

$$\overline{T}_{(x)} = \frac{1}{N} \sum_{n=0}^{N} T_{(x_n)}, \; 0 \leqslant x_n \leqslant x. \tag{39}$$

These curves and appropriate analysis show that in the case of a linear wall energy flux distribution, a lower average (and maximum) wall temperature is obtained and a larger wall area is needed to convect the same amount of energy than in the case of an exponential distribution.

Table 4. Comparison for $q = (1+x)$

x	$T_{Z(0)}$	$T_{\text{ref}(0)}$	$T_{Ka(0)}$
0	0	0	0
0.2	1.6759	1.6759	1.6717
0.4	2.1240	2.1226	2.1294
0.6	2.5150	2.5101	2.5215
0.8	2.8829	2.8773	2.8904
1.0	3.2412	3.2327	3.2518
1.7	4.4559	4.4440	4.4648
2.1	5.1377	5.1239	5.1480
2.5	5.8166	5.7970	5.8205
3.1	6.8324	6.8047	6.8324

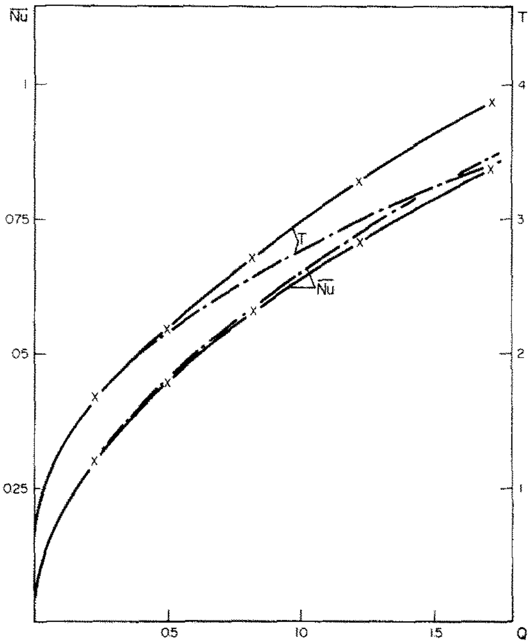

FIG. 3. Comparison of the average Nusselt number and the wall temperature variations in function of the amount of energy convected, for the exponential wall flux distribution —— and the linear distribution $-\cdot-\cdot-$.

CONCLUSIONS

It is clear that the calculation of σ and ω are much easier than equivalent transformations used in other nonsimilar solution methods.

In the case of the geometries treated in this paper, g_x is constant with respect to x; thus the values of ε_n do not depend on the inclination angle. Since the solution of system (16)–(19) depends only on ε_n, F_n and θ_n but does not depend on γ, only σ and ω vary with it. The reason of this result is that both p and ε are independent with respect to the transformation constants.

Another advantage is the easiness of calculating $\varepsilon_n s$, e.g.:

(a) for the uniform flux distribution, $p = 0$, $\varepsilon = 0$, i.e. only the zeroth-order is to be solved;

(b) for all the other cases the uniform flux distribution solution can be used as the zeroth-order's, which saves a lot of computing time.

The proposed method is limited by the apparition of instability in the flow, which is, as was pointed out in the Introduction, predicted only by experimental or adequate analytical study.

The fact that we neglected the normal-direction Navier–Stokes equation may be criticized and may explain the errors in the generalization, especially in the large inclination angles.

The comparison between our results and those given by the literature shows that our solution method, which is shorter, gives a better precision.

It would be interesting to treat other flux distributions and geometry configurations. A general method to determine the adequate transformations for every specific problem is given in [39] where the details of the results given in this paper are discussed.

Acknowledgement—The authors would like to express their acknowledgement to Dr M. S. Abdelhafiez, Director of the Laboratory of Solar Energy Applications, Faculty of

Engineering, Alexandria University, Alexandria, Egypt for his precious contribution in the preparation of this paper during and after his visit to the University of Perpignan.

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